Math 33A :

HW 1 Released!

Due 7/8 (Monday) @ 11:59 pm

OH : Mon, Thurs 1-2 pm Zoom

Worksheet for this Week on BruinLearn

& Webpage

Invertibility:

A $n \times m$ matrix, A^{-1} is the inverse of A (if it exists)

if
$$\left.\begin{aligned} AA^{-1} &= I \\ A^{-1}A &= I \end{aligned}\right\} \text{identity matrix} \quad \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix}$$

When does A^{-1} exist:

1. A has to be square $(n \times n)$ # of rows = # of cols.

2. $\text{rank}(A) = n$ $\quad (\ker(A) = \vec{0})$

$$\text{rref}(A) = \begin{bmatrix} 1 & \cdots & 0 \\ 0 & & 1 \end{bmatrix}$$

If both are true, then A is <u>invertible</u> and an inverse A^{-1} exists.

How to find A^{-1}? $(A \ n \times n)$

$$\left[\begin{array}{c|c} A & I_n \end{array}\right] \xrightarrow[\text{reduce to rref}]{\text{row}} \left[\begin{array}{c|c} I_n & A^{-1} \end{array}\right]$$

Ex: Determine if $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{bmatrix}$ is invertible, and if

if is, find A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 1 & 3 & 6 & 0 & 0 & 1 \end{array}\right] - (I) \rightsquigarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 1 & 0 \\ 1 & 3 & 6 & 0 & 0 & 1 \end{array}\right] - (I) \rightsquigarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 1 & 0 \\ 0 & 2 & 5 & -1 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \\ -2(\mathrm{II}) \end{array} \rightsquigarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{array}\right] \begin{array}{l} \\ -2(\mathrm{III}) \\ \\ \end{array} \rightsquigarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -3 & 5 & -2 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{array}\right] \begin{array}{l} -(\mathrm{III}) \\ \\ \\ \end{array} \rightsquigarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & 2 & -1 \\ 0 & 1 & 0 & -3 & 5 & -2 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{array}\right] - (\mathrm{II}) \quad \rightsquigarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -3 & 1 \\ 0 & 1 & 0 & -3 & 5 & -2 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{array}\right]$$

$$\underbrace{\begin{matrix} 3 & -3 & 1 \\ -3 & 5 & -2 \\ 1 & -2 & 1 \end{matrix}}_{A^{-1}}$$

$$\underset{A}{\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{bmatrix}} \cdot \underset{A^{-1}}{\begin{bmatrix} 3 & -3 & 1 \\ -3 & 5 & -2 \\ 1 & -2 & 1 \end{bmatrix}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3-3+1 & -3+5-2 & 1-2+1 \\ 3-6+3 & -3+10-6 & 1-4+3 \\ 3-9+6 & -3+15-12 & 1-6+6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$A\vec{x} = \vec{b}$

coefficient matrix

augmented col.

$$\left[\begin{array}{c|c} A & \vec{b} \end{array}\right]$$

If A invertible, multiply both sides by A^{-1}:

$$(A^{-1}A)\vec{x} = A^{-1}\vec{b} \quad \Rightarrow \quad \vec{x} = \underbrace{A^{-1}\vec{b}}$$

$$I\vec{x} = A^{-1}\vec{b}$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{bmatrix} \vec{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$x + y + z = 1$$

$$x + 2y + 3z = 0$$

$$x + 3y + 6z = 0$$

$$A^{-1} = \begin{bmatrix} 3 & -3 & 1 \\ -3 & 5 & -2 \\ 1 & -2 & 1 \end{bmatrix}$$

$$\vec{x} = \begin{bmatrix} 3 & -3 & 1 \\ -3 & 5 & -2 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ 1 \end{bmatrix}$$

$$\boxed{\begin{aligned} x &= 3 \\ y &= -3 \\ z &= 1 \end{aligned}}$$

1.5: (a) There exists an invertible $n \times n$ matrix w/ two identical rows

$\boxed{\text{False}}$

$$\left.\begin{bmatrix} \text{---} & \vec{a}_1 & \text{---} \\ \text{---} & \vec{a}_2 & \text{---} \\ & \vdots & \\ \text{---} & \vec{a}_n & \text{---} \end{bmatrix}\begin{array}{l} \big\} \text{ identical} \\ -(I) \\ \\ \\ \end{array}\right. \xrightarrow{\text{row reduction}} \begin{bmatrix} \text{---} & \vec{a}_1 & \text{---} \\ 0 & \cdots & 0 \\ & \vdots & \\ \text{---} & \vec{a}_n & \text{---} \end{bmatrix}$$

When you have a row of all 0's, <u>not</u> invertible

b/c $\operatorname{rank}(A) < n$

1.5 (b) There exists an invertible 2×2 matrix A s.t.

$$\underbrace{A^{-1} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}}_{\text{not invertible}}$$

False b/c this is not invertible

$$(A^{-1})^{-1} = A \implies$$ A^{-1} is also invertible w/ inverse A

A, B are both invertible, then $(AB)^{-1} = B^{-1}A^{-1}$

$(A+B)^{-1}$?

$A+B$ might not be invertible !!!

ex: A invertible, so is $-A$

$A + -A = \underline{0}$ not invertible